APPENDIX A

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% BEGINNING OF PSEUDO CODE

% compute scale factor A, and time constants a, b from physical system
% parameters

A = Vmax * Kt / (Re * Rm + Kt * Kb) * l * k;

p1  = 1/Jm/Ie * (-Ie * Rm - Re * Jm + sqrt(Ie^2 * Rm^2 - 2 * Re * Rm * Ie
        * Jm + Re^2 * Jm^2 - 4 * Kt * Kb * Ie * Jm)) / 2;
p2  = 1/Jm/Ie * (-Ie * Rm - Re * Jm - sqrt(Ie^2 * Rm^2 - 2 * Re * Rm * Ie
        * Jm +  Re^2 * Jm^2 - 4 * Kt * Kb * Ie * Jm)) / 2;

a = max(-p1,-p2)
b = min(-p1,-p2)

% make initial guesses for step durations

et1 = 1;
et2 = .005;
et3 = 1;

% set maximum iteration count

Nmax = 1000;

for j = 1:Nmax
        % save old values of step time intervals
        et3old = et3;
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        et2old = et2;
        et1old = et1;

    % iterate for switch times using fixed voltage level Vmax

        et3 = -log(1.0 / 2.0 - exp(-et1 * a) / 2 + exp(-et2 * a)) / a;
        et2 = 1/b * log(2.0) + 3 * et3 - 1/b * log(2 * exp(1/A * b * X) *
              exp(et3 * b) - sqrt( 4.0) * sqrt(exp(1/A * b * X)) * exp(et3 * b)
              * sqrt(exp(1/A * b * X)+exp(et3 * b)^2 - 2 * exp(et3 * b)));
        et1 = - (-2 * A * et2 + 2 * A * et3 - X) / A;

        if norm([et3old - et3 et2old - et2 et1old - et1], inf) <= eps * 2
              break
        end
        if j==Nmax
              error(['error - failure to converge after ', num2str(Nmax),'
              iterations'])
        end
    end

    % round up pulse duration to nearest sample interval,
    % convert to intervals between steps to make sure that voltage
    % requirements will not increase (beyond Vmax).

    dt1=ceil((et1 - et2) / dt) * dt;
    dt2=ceil((et2 - et3) / dt) * dt;
    dt3=ceil((et3) / dt) * dt;

    et123 = [et1, et2, et3]
        % convert back to total step duration.
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et1 = dt1 + dt2 + dt3;

et2 = dt2 + dt3;

et3 = dt3;

% In the following, the original constraints equations involving XF1, XF2, % and XF3 have been modified to include a variable voltage level applied at % each step (instead of the fixed maximum (+/-) Vmax).

% The original equations for XF1, XF2, and XF3 follow:

% $XF_1(t_{end}) = V_0F_1(t_{tend} - t_0) - 2V_0F_1(t_{end} - t_1) + 2V_0F_1(t_{end} - t_2)$

% $XF_2(t_{end}) = V_0F_2(t_{tend} - t_0) - 2V_0F_2(t_{end} - t_1) + 2V_0F_1(t_{end} - t_2)$

% $XF_3(t_{end}) = V_0F_3(t_{tend} - t_0) - 2V_0F_2(t_{end} - t_1) + 2V_0F_1(t_{end} - t_2)$

% And the modified equation including adjustable relative levels of voltage % L1, L2 and L3 are:

% $XF_1(t_{end}) = L_1V_0F_1(t_{tend} - t_0) - L_2V_0F_1(t_{end} - t_1) + L_3V_0F_1(t_{end} - t_2)$

% $XF_2(t_{end}) = L_1V_0F_2(t_{tend} - t_0) - L_2V_0F_2(t_{end} - t_1) + L_3V_0F_1(t_{end} - t_2)$

% $XF_3(t_{end}) = L_1V_0F_3(t_{tend} - t_0) - L_2V_0F_2(t_{end} - t_1) + L_3V_0F_1(t_{end} - t_2)$

% And the corresponding constraint equations are:

% $XF_1(t_{end}) = \text{Finalpos}$

% $XF_2(t_{end}) = 0$

% $XF_3(t_{end}) = 0$

% Where all of the times indicated have discrete values, e.g. corresponding to % the controller update rate.

% It should be noted that after the digital switch times are fixed, the constraint % equations derived from the equations above form a linear set of equations in % the unknown relative voltage levels L1, L2 and L3 and any standard linear

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% method can be used to solve for the relative voltage levels. In the equations
% for (L1, L2 and L3) that follow, the solution was obtained by algebraic
% means (and are not particularly compact.)

% compute new relative voltage step levels
% L1, L2 and L3 are nominally assigned to "1", "-2" and "+2", respectively

s1  =  X * (exp(-et3 * b) * exp(-et2 * a) + exp(-et3 * a) + exp(-et2 *b ) -
       exp(-et2 *b) * exp(-et3 * a) - exp(-et2 * a) - exp(-et3 *b ));
s2  =  1 / (et2 * exp(-et1 * b) * exp(-et3 * a) + exp(-et2 * b) * et3 *
       exp(-et1 * a)- et2 * exp(-et3 * a) - et2 * exp(-et1 * b)  -et3 *
       exp(-et1 * a) - exp(-et2 * b) * et3+exp(-et3 * b) * et1 * exp(-et2 * a)
       + exp(-et3 * a) * et1 + exp(-et2 * b) * et1 - exp(-et2 * b) * et1 *
       exp(-et3 * a) - et3 * exp(-et1 * b) * exp(-et2 * a) - exp(-et2 * a) * et1 -
       exp(-et3 * b) * et1 - exp(-et3 * b) * et2 * exp(-et1 * a) + et3 *
       exp(-et1 * b) + et2 * exp(-et1 * a) + exp(-et3 * b) * et2 + et3 *
       exp(-et2 * a)) / A;

L1  =  s1 * s2;

s1  =  1 / (et2 * exp(-et1 * b) * exp(-et3 * a) + exp(-et2 * b) * et3 *
       exp(-et1 * a) - et2 * exp(-et3 * a) - et2 * exp(-et1 * b) - et3 *
       exp(-et1 * a) - exp(-et2 * b) * et3 + exp(-et3 * b) * et1 *
       exp(-et2 * a) + exp(-et3 * a) * et1+exp(-et2 * b) * et1 -
       exp(-et2 * b) * et1 * exp(-et3 * a) - et3 * exp(-et1 * b) *
       exp(-et2 * a) - exp(-et2 * a) * et1 - exp(-et3 * b) * et1 - exp(-et3 * b) *
       et2 * exp(-et1 * a) + et3 * exp(-et1 * b) + et2 * exp(-et1 * a) +
       exp(-et3 *b ) * et2 + et3 * exp(-et2 * a)) * X;
s2  =  (exp(-et2 * b) * exp(-et1 * a) - exp(-et1 * a) - exp(-et2 * b) -
       exp(-et1 * b) * exp(-et2 * a) + exp(-et1 * b) + exp(-et2 * a)) / A;
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L3 = s1*s2;

s1 = exp(-et1 * a) - exp(-et3 * a) + exp(-et3 * b) - exp(-et1 * b) -
     exp(-et3 * b) * exp(-et1 * a) + exp(-et1 * b) * exp(-et3 * a);
s2 = X / (et2 * exp(-et1 * b) * exp(-et3 * a) + exp(-et2 * b) * et3 *
     exp(-et1 * a) - et2 * exp(-et3 * a) - et2 * exp(-et1 * b) - et3 *
     exp(-et1 * a) - exp(-et2 * b) * et3 + exp(-et3 * b) * et1 * exp(-et2 * a)
     + exp(-et3 * a) * et1 + exp(-et2 * b) * et1 - exp(-et2 * b) * et1 *
     exp(-et3 * a) - et3 * exp(-et1 * b) * exp(-et2 *a ) - exp(-et2 * a) * et1-
     exp(-et3 * b) * et1 - exp(-et3 * b) * et2 * exp(-et1 * a) + et3 *
     exp(-et1 * b) + et2 * exp(-et1 * a) + exp(-et3 * b) * et2 + et3 *
     exp(-et2 * a)) / A;

L2 = s1 * s2;

% convert accumulated voltage steps to sequential voltage level
V1 = Vmax * (L1);
V2 = Vmax * (L1 + L2);
V3 = Vmax * (L1 + L2 + L3);

% END OF PSEUDO CODE
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